DESCRIPTION OF THE COURSE OF STUDY

| Course code | 0541.6.MAT2.C.GA |  |
| :--- | :---: | :---: |
| Name of the course in | Polish | Geometria algebraiczna |
|  | English | Algebraic Geometry |

## 1. LOCATION OF THE COURSE OF STUDY WITHIN THE SYSTEM OF STUDIES

| 1.1. Field of study | Mathematics |
| :--- | :--- |
| 1.2. Mode of study | full-time studies |
| 1.3. Level of study | Graduate (Master) |
| 1.4. Profile of study* | general academic profile of studies |
| 1.5. Person/s preparing the course description | dr Mateusz Masternak |
| 1.6. Contact | mateusz.masternak @ujk.edu.pl |

2. GENERAL CHARACTERISTICS OF THE COURSE OF STUDY

| 2.1. Language of instruction | English and Polish |
| :--- | :--- |
| 2.2. Prerequisites* | basics of algebra |

## 3. DETAILED CHARACTERISTICS OF THE COURSE OF STUDY

| 3.1. Form of classes |  | lectures and classes |
| :---: | :---: | :---: |
| 3.2. Place of classes |  | classes in the UJK teaching room |
| 3.3. Form of assessment |  | Exam (lectures), graded credit (classes) |
| 3.4. Teaching methods |  | Information lecture, discusions, workshop, solving problems. |
| 3.5. Bibliography | Required reading | 1. William Fulton, Algebraic Curves, An Introduction to Algebraic Geometry, 2008, which is available for free (legally) here: <br> http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf ; <br> 2. Igor R. Shafarevich, Basic Algebraic Geometry 1, Varieties in Projective Space, Third Edition, Springer-Verlag Berlin Heidelberg, 2013; <br> 3. Robin Hartshorne, (Graduate Texts in Mathematics), Algebraic geometry, Springer, 1977; |
|  | Further reading | 1. Otto Forster, Lectures on Riemann Surfaces, Springer, New York, 1999; <br> 2. Armin Rainer, Introduction to Riemann Surfaces, Lecture Notes, 2018, which is available for free (legally) here: <br> https://www.mat.univie.ac.at/~armin/lect/Riemann_surfaces.pdf ; <br> 3. David Mumford, Algebraic geometry I: Complex projective varieties, Classics in Mathematics, Springer-Verlag, 1995; <br> 4. Egbert Brieskorn, Horst Knörrer, Plane Algebraic Curves, (translated by John Stillwell), Birkhäuser, Basel, 2012. |

## 4. OBJECTIVES, SYLLABUS CONTENT AND INTENDED LEARNING OUTCOMES

### 4.1. Course objectives

The subject is an introduction to algebraic geometry. During the course, affine algebraic sets will be presented and the basics of algebraic projective geometry and properties of algebraic varieties will be discussed. In particular, elements of algebraic curves theory will be considered in more detail.

## Lecture

C1 - learning about affine algebraic sets,
C 2 - learning the basics of algebraic projective geometry and properties of algebraic varieties
C3-presentation of the basic theory of algebraic curves.
Classes
C1 - mastering the ability to study the geometric properties (and arithmetic properties) of algebraic manifolds with particular emphasis on the study of the properties and description of algebraic curves.
C2 - to develop the habit of learning, improving one's own work, and formulating questions that serve to deepening one's own understanding of a given topic.

### 4.2. Detailed syllabus

## Lecture:

Affine algebraic sets and Zariski topology. Hilbert Zeros Theorem (Nullstellensatz). Affine varieties over an algebraically closed field. Regular mappings. Irreducible sets. Homogeneous polynomials. Projective varieties. Affine plane and projective plane. Regular maps of projective subsets and rational maps. Plane curves. Local properties of curves. Projective plane curves. Intersection numbers. Bézout Theorem. Resolution of singularities (quadratic transformations, blowing up). Riemann surfaces. Divisors. Sheaves. Cohomologies. Riemann-Roch Theorem. Hurvitz Theorem. Elliptic curves.

## Seminar sessions:

Testing of geometrical (and arithmetical) properties of algebraic sets, in particular, complex plane curves will be considered in more detail. Testing of local properties of curves and their singularities. The intersection multiplicity and its properties. Determination of
the intersection multiplicity of curves. Puiseux Theorem. Branch. Number of branches. Singularity invariants. Milnor number and its calculation. The Newton diagram and the Newton polygon. Estimations of the intersection multiplicity and the number of branches and the Milnor number in terms of the Newton diagrams. Information about the Newton algorithm. Information about the semi-group of a branch. Kouchnirenko Theorems (local and global version). Berntstein Theorem as a reinforcement of Bézout Theorem.

### 4.3 Intended learning outcomes

| ¢ٌ8080 | A student, who passed the course | Relation to learning outcomes |
| :---: | :---: | :---: |
| within the scope of KNOWLEDGE: |  |  |
| W01 | understands well the role and importance of mathematical reasoning in algebraic geometry relating to the study and description of properties of algebraic sets and manifolds. | MAT2A_W01 MAT2A_W02 MAT2A W11 |
| W02 | knows the most important concepts, theorems and hypotheses in the field of the foundations of algebraic geometry with particular emphasis on the theory of flat algebraic curves. | MAT2A_W01 MAT2A_W02 MAT2A W11 |
| W03 | knows examples of applications of algebraic methods (in possible combination with the use of tools from other branches of mathematics, inter alia in topology, algebraic topology, complex analysis and differential geometry) in solving problems in the field of algebraic geometry (including solving practical tasks). | MAT2A_W01 MAT2A_W02 MAT2A_W11 |
| within the scope of ABILITIES: |  |  |
| U01 | can construct reasoning and proof of the basics of algebraic geometry and can select counterexamples to refute erroneous hypotheses in this area, as well as the ability to verify the correctness of the reasoning in the formal proofs of theorems in its field. | MAT2A_U01 MAT2A U03 MAT2A U10 |
| U02 | can see and distinguish formal structures related to the basic objects studied within algebraic geometry and understands the importance of these structures. | $\begin{aligned} & \hline \text { MAT2A_U01 } \\ & \text { MAT2A_U03 } \\ & \text { MAT2A_U10 } \end{aligned}$ |
| U03 | is able to carry out theorem proofs from the basics of algebraic geometry, in which he uses and combines, if necessary, also techniques from other branches of mathematics, inter alia from the field of commutative algebra, topology, algebraic topology, complex analysis and differential geometry. | $\begin{aligned} & \hline \text { MAT2A_U01 } \\ & \text { MAT2A_U03 } \\ & \text { MAT2A_U10 } \end{aligned}$ |
| within the scope of SOCIAL COMPETENCE: |  |  |
| K01 | recognizes the importance of knowledge of the basics of algebraic geometry in solving theoretical and practical problems. | MAT2A_K02 |

### 4.4. Methods of assessment of the intended learning outcomes

| Teaching outcomes (code) | Method of assessment (+/-) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Exam oral/written* |  |  | Test* |  |  | Project* |  |  | Effort in class* |  |  | Self-study* |  |  | Group work* |  |  | Others* e.g. standardized test used in elearning |  |  |
|  | Form of classes |  |  | Form of classes |  |  | Form of classes |  |  | Form of classes |  |  | Form of classes |  |  | Form of classes |  |  | Form of classes |  |  |
|  | $L$ | C | ... | $L$ | C | ... | $L$ | C | ... | $L$ | C | ... | $L$ | C | ... | $L$ | C | ... | $L$ | C | . |
| W01 | + |  |  |  |  | + |  |  |  | + |  | + | + |  | + |  |  |  |  |  |  |
| W02 | + |  |  |  |  | + |  |  |  | + |  | + | + |  | + |  |  |  |  |  |  |
| W11 | + |  |  |  |  | + |  |  |  | + |  | + | + |  | + |  |  |  |  |  |  |
| U01 |  |  |  |  |  | + |  |  |  | + |  | + | + |  | + |  |  |  |  |  |  |
| U03 |  |  |  |  |  | + |  |  |  | + |  | + | + |  | + |  |  |  |  |  |  |
| U10 |  |  |  |  |  | + |  |  |  | + |  | + | + |  | + |  |  |  |  |  |  |
| K01 | + |  |  |  |  | + |  |  |  | + |  | + | + |  | + |  |  |  |  |  |  |

*delete as appropriate
4.5. Criteria of assessment of the intended learning outcomes

| Form of classes | Grade | Criterion of assessment |
| :---: | :---: | :---: |
|  | 3 | at least $50 \%$ and no more than $60 \%$ of the total number of points possible |
|  | 3,5 | more than $60 \%$ and no more than $70 \%$ of the total number of points possible |
|  | 4 | more than $70 \%$ and no more than $80 \%$ of the total number of points possible |
|  | 4,5 | more than $80 \%$ and no more than $90 \%$ of the total number of points possible |
|  | 5 | more than $90 \%$ of the total number of points possible |


|  | 3 | at least $50 \%$ and no more than $60 \%$ of the total number of points possible |
| :---: | :---: | :---: |
|  | 3,5 | more than $60 \%$ and no more than $70 \%$ of the total number of points possible |
|  | 4 | more than $70 \%$ and no more than $80 \%$ of the total number of points possible |
|  | 4,5 | more than $80 \%$ and no more than $90 \%$ of the total number of points possible |
|  | 5 | more than $90 \%$ of the total number of points possible |

## 5. BALANCE OF ECTS CREDITS - STUDENT'S WORK INPUT

| Category | Student's workload |  |
| :--- | :---: | :---: |
|  | Full-time <br> studies | Extramural studies |
| NUMBER OF HOURS WITH THE DIRECT PARTICIPATION OF THE TEACHER <br> /CONTACT HOURS/ | $\mathbf{4 7}$ |  |
| Participation in lectures* | 15 |  |
| Participation in classes, seminars, laboratories* | 30 |  |
| Preparation in the exam/ final test* | 2 |  |
| INDEPENDENT WORK OF THE STUDENT/NON-CONTACT HOURS/ | $\mathbf{2 8}$ |  |
| Preparation for the lecture* | 8 |  |
| Preparation for the classes, seminars, laboratories* | 10 |  |
| Preparation for the exam/test* | 10 |  |
| TOTAL NUMBER OF HOURS | $\mathbf{7 5}$ |  |
| ECTS credits for the course of study | $\mathbf{3}$ |  |

## *delete as appropriate

Accepted for execution (date and legible signatures of the teachers running the course in the given academic year)

