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Mini-Workshop: Singularities in Kielce, May 22, 2026

Program

11:00 - 11:05	Greeting
11:05 - 11:50	*A Eudoxian study of discriminant curves*, Evelia R. García Barroso
11:50 - 12:00	Coffee break
12:00 - 12:45	*The jump of Milnor number in linear deformations with fixed order*, Aleksandra Zakrzewska
12:45 - 13:00	Coffee break
13:00 - 13:45	*The Real Jacobian Conjecture for map with one component having degree 6*, Janusz Gwoździewicz
13:45 - 13:55	Coffee break
13:55 - 14:25	*Local complex curve with prescribed Eggers-Płoski tree*, Andrzej Lenarcik
15.00	Lunch, Restaurant "Pod Złotą Różą", Plac Moniuszki 7, (in the center of Kielce)

Abstracts

A Eudoxian study of discriminant curves

Evelia R. García Barroso
Universidad de La Laguna, Tenerife, España.

I will present joint work with Patrick Popescu-Pampu. Let

$$(f, g) : (S, s) \to (\mathbb{C}^2, 0)$$

be a finite morphism from the germ of a normal complex analytic surface to the germ of the complex affine plane at the origin. We prove that the affine algebraic curve in $\mathbb{C}^2$ determined by the initial Newton polynomial of a defining equation of the discriminant germ of (f, g) is, up to toric automorphisms, completely determined by the curve germs defined by f and g. This extends a theorem of Gryszka, Gwoździewicz and Parusiński, corresponding to the case where (S, s) is smooth.

The jump of Milnor number in linear deformations with fixed order

Aleksandra Zakrzewska
(joint results with Tadeusz Krasiński)
University of Łódź, Poland

Let f_0 be a plane curve singularity. We compute the jump of Milnor number of a homogeneous f_0 of order n in the class of linear deformations $f_s = f_0 + sg$ with fixed order $\mathrm{ord} f_s = j < n$, i.e. $\mathrm{ord} g = j$.

The Real Jacobian Conjecture for map with one component having degree 6

Janusz Gwoździewicz
University of the National Education Commission, Kraków, Poland

We show that if $F = (p, q) : \mathbb{R}^2 \to \mathbb{R}^2$ is a polynomial map such that the degree of p is 6 and whose Jacobian determinant is nowhere zero, then F is injective. This together with previous works in the literature, guarantees the validity of the real Jacobian conjecture in the plane provided that one of the coordinate functions of the map has degree smaller than 7.

Local complex curve with prescribed Eggers-Płoski tree

Andrzej Lenarcik
Kielce University of Technology, Poland

We consider the ring of convergent power series of two complex variables. A series is called singular if it vanishes at zero with first-order partial derivatives. We consider isolated singularities (corresponding to reduced series). Every singularity decomposes into a finite number of irreducible series (branches). The topological type of a singularity depends on the characteristics of the branches and their mutual intersection multiplicities. Eggers [1] represented the singularity type by using a decorated graph constructed from Puiseux series of branches (see also [4]). An analogous construction, applying Płoski's logarithmic distance [2, 3] - without referring to any coordinate system - was presented in [5, 6, 7]. The resulting tree has, as in the construction of Eggers, two types of vertices (black and white) and two types of edges (solid and discontinuous). Furthermore, to every black vertex a rational number (logarithmic distance) is assigned as a decoration. During the lecture, the graphs that can be Eggers-Płoski trees of singularities will be characterized.

References

[1] H. Eggers, *Polarinvarianten und die Topologie von Kurvensingularitäten*, Bonner Math. Schriften 147, Universität Bonn, Bonn 1982

[2] A. Płoski, *Remarque sur la multiplicité d'intersection des branches planes*, Bull. Pol. Acad. Sci. Math. 33(11-12), (1985) 601–605.

[3] J. Chądzyński, A. Płoski, *An inequality for the intersection multiplicity of analytic curves.* Bull. Polish Acad. Sci. Math. 36 (1988), no.3-4, 13–17.

[4] E. R. García Barroso, *Sur les courbes polaires d'une courbe plane réduite*, Proc. London Math. Soc., III, 81,1 (2000), 1–28.

[5] A. Lenarcik, *On the Łojasiewicz exponent, special direction and maximal polar quotient*, arXiv:1112.5578v1, Dec 2011

[6] A. Lenarcik, *Eggers tree and jacobian Newton polygon*, Manuscripta Math. 142 (2013), 233-244.

[7] A. Lenarcik, *On the jacobian Newton polygon, branches and dandelions*, Ann. Polon. Math. 135 (2025), 221–236.